University Learning in Schools

Mathematics

3D Geometry in the real world

Lesson Plans

The Brilliant Club

Department for Education

SUPPORTED BY MAYOR OF LONDON

Before the block of lessons begin, we plan on giving each pupil a "Vocabulary/Knowledge Rating" sheet (see the collection of resources). This should be given out a few weeks before the start of this module and collected in before the first proper lesson, to assess the pupils' prior knowledge of some key vocabulary. It will also enable us to create better groupings for the subsequent group work in the six lessons.

The five keywords that we will be looking at via this sheet are: symmetry, nets, faces, edges, vertices

Lesson 1 – Footballs
Learning Objectives: • Review of some terminology when talking about 2D and 3D shapes, namely the definitions of 2D, 3D, polygons, nets, polyhedra, symmetry, protractor, compass. • Review of how to construct a 3D shape from its net, by gluing together appropriate edges that have flaps, and folding the resulting structure together.
Resources: • Concept map sheet, for the keywords given in the learning objectives • Printed net of a 3D football (comes in 6 separate parts or 1 part). • Printed net of a 3D truncated cuboctahedron (comes in 1 part), if needed. • WWW/EBI sheet
Activities: **First activity:** pairwise/mini-group discussion of the keywords listed above, which will be turned into a concept map, using full sentence descriptions and/or pictures to demonstrate their meaning. **Second activity:** full class discussion of the keywords and review of the concept maps, with an emphasis on **nets**, which bridge the 2D and 3D world. **Final activity**: construct the 3D football from its net (full instructions/help can be given if needed, but the pupils will be expected to attempt this unaided at first). This may be split into two versions: for most pupils, there is a football net on one sheet of A4 paper, but for more confident ones, they can use the version of the football net that spans six separate A4 pages (this requires some thought as to how the pieces glue together). **Extension (for those who finish early):** additional nets of more complicated 3D shapes (like the cuboctahedron mentioned above) can be given out to the pupils who finish early. These can then be assembled into their 3D shapes, and finished at home if

needed.

Homework: the pupils will be expected to write a short report about 3D shapes in nature/the real world. There are lots of potential themes here, such as architecture (pyramids of Giza…), plants/animals (e.g. bees make hexagonal honeycomb structures), and man-made objects (triangular structures/boxes, which are very strong, cuboid boxes that tessellate…).

Pupil Assessment (include approach and expected outcome):

The "Vocabulary/Knowledge Rating" sheets should be examined before the first lesson, to gain some understanding of the level that the pupils are at, and to better inform groupings for future lessons.

The concept map will be reviewed via a full-class discussion, to allow the pupils to add any missing details to their concept maps. This will contribute to the baseline assessment.

The 3D footballs will be expected to be completed at home (if not within the lesson). Of course, the pupils should keep a log of all the steps taken to construct the football.

At the end of the lesson, there will be a quick WWW/EBI sheet in pairs, which will contribute to a continuing log of the whole module. This allows for feedback from everyone involved (teacher/pupils/observers).

The homework will be reviewed at the beginning of the second lesson in a short discussion.

Lesson Plan

Lesson 2 – The cube
Learning Objectives: • Further recall of how to construct a 3D shape from its net, this time with the cube (an activity that may have already been done before) • Review of more keywords when talking about 3D shapes, especially the definitions of vertices, edges, faces, symmetry, and rotation of order n.
Resources: • Printed net of a 3D cube (comes in 1 part). • "Vocabulary/Knowledge Rating sheet", to be expanded into a full concept map • WWW/EBI sheet
Activities: **Beginning of lesson:** paired discussion of homework, to then be brought into a full-class discussion of the key facts (i.e. 3D shapes used in the real world) – related to the content of lesson 5. Also bring in the key vocabulary: vertices, edges, faces. **Middle of lesson, part 1:** pupils are each given a net of a 3D cube to fold it into a cube and glue the edges. The beginning of the worksheet with information about vertices/edges/faces can start to be filled in. **Middle of lesson, part 2:** once all pupils have constructed the cube (with help from others if necessary), they can review the three keywords: vertices, edges and faces. They can count each of these for the cube, as well as the football from lesson 1, and verify Euler's formula (#Vertices – #Edges + #Faces = 2). **Remainder of lesson:** give a recap of rotational symmetry of 2D shapes, and if possible an introduction to 3D rotations of order n, using the cube. These rotations can be demonstrated if it is practical to do so (e.g. wooden stick through a diagonal of the cube, and then spin the stick by 120 or 240 degrees)… **Homework:** make a list of as many 3D shapes as possible (i.e. the pupils can do a Google search and find lots of examples), giving the properties/real--world uses of each one. There will be a small prize awarded to the pupil who finds the most/compiles the most comprehensively-detailed list.
Pupil Assessment (include approach and expected outcome): In the discussion of the homework, it is hoped that each pupil will be able to come up

with a different example of a 3D shape in nature.

There will be a short paired discussion after the cube has been constructed and its properties (number of vertices, edges, faces) found.

Finally, there will be a WWW/EBI evaluation sheet at the end of the lesson.

Lesson Plan

Lesson 3 – Square-based pyramids and octahedra
Learning Objectives: • There will be further reinforcement of the skill of turning a 2D net into a 3D object • The key vocabulary of vertices, edges, faces and rotational symmetry will be recapped, especially the concept of rotational symmetry using the examples of the square-based pyramid and octahedron.
Resources: • Printed net of a 3D square-based pyramid • Printed net of a 3D octahedron • WWW/EBI sheet
Activities: **Beginning of lesson:** class discussion of homework, going through different shapes and their properties (and the pupils can try to group them as regular platonic solids, Archim[p-[edean solids etc.). Then highlight two of them: the square-based pyramid and the octahedron. Other language aspects can be brought in, e.g. the prefix “oct-“ which means 8 (as in octopus, octagon, octogenarian, October…) **Middle of lesson:** the pupils are put into pairs (the earlier concept maps and vocabulary/knowledge rating sheets will be used to guide this process). One pupil in each pair will be given the net of a square-based pyramid, and the other will receive the net of an octahedron. Everyone will fold their net into the correct shape, recording its properties and (with help if needed) discover the rotational symmetry it possesses. **End of lesson:** once all pupils have constructed their shape and recorded the properties, they can get back into pairs to discuss their shapes, comparing and contrasting them. In particular, they can explain the rotational symmetry that each shape possesses! After this, a WWW/EBI sheet can be completed for lesson 3. **Homework (for lesson 4):** bring in a net of a 3D object, or an interesting 3D shape, to deconstruct for lesson 4.
Pupil Assessment (include approach and expected outcome): In the discussion of the homework, it is hoped that each pupil will be able to come up with a different 3D shape, and in particular cover all the (regular) platonic solids, as well as some of the weird and wonderful Archimedean solids. Essentially, the bigger the variety, the better! The short discussion on language aspects can be extended if chosen (e.g. other prefixes like with tri, pent, hex, sept/hept…) or even incorporated into the homework of

lesson 2 if the teacher feels like this would add to the work.

The paired work in constructing the square-based pyramid and octahedron gives some more opportunity for peer assessment, as does the WWW/EBI sheet.

Lesson Plan

Lesson 4 – Shapes and nets from home
Learning Objectives: • This lesson has the potential to introduce pupils to a wide range of nets/3D shapes that they are completely unfamiliar with. • Pupils are encouraged to fall back on the key vocabulary of vertices, edges, faces and rotational symmetry to describe the shape, and comparing these properties with those of some known shapes (football, cube, square-based pyramid, octahedron) that have been previously explored. • The wider context of this lesson is that it will give inspiration to the 3D shape that will be designed and created during lesson 5.
Resources: • Spare nets/Spare 3D shapes that are interesting to study • WWW/EBI sheet
Activities: **Beginning of lesson:** there will be a class discussion of some of the objects/nets brought in by the pupils, so that everyone can see what everyone else has brought in (or chosen from our extra collection). **Middle of lesson:** the pupils will be put into pairs, hopefully with one having a net to look at and the other having a 3D object. The net can be constructed into its 3D shape, whilst the pupils could also try to work out what a suitable net for the 3D object is. During this time, the usual properties (vertices, edges, faces, rotational symmetry) can be explored. **End of lesson:** At the same time as the WWW/EBI sheet is completed, the pupils will be encouraged to leave their findings open next to their shape and net. This will allow other pupils to see each other's work, but also give them some more opportunity to prepare for lesson 5 (design and create a 3D shape!). **Homework:** investigate 3D printers. How do they work? Compare and contrast with 2D printers (both inkjet and laser ones). The unglamorous title for this homework is "a history of printers", but the technology behind them is more exciting than the topic would at first suggest!
Pupil Assessment (include approach and expected outcome): The paired work enables continuous peer assessment; if pupils finish early, then it is possible to get pairs to present their findings to each other and get some presentation practice in a smaller setting.

Lesson Plan

Lesson 5 – Creating and designing a new 3D shape
Learning Objectives: • The main objective is to have (at least the draft of) a design of a new 3D shape
Resources: • Several sheets of A3 paper for the rough (and later neat versions) of the designs of the 3D shapes
Activities: **Beginning of lesson:** brief recap of homework – emphasise that all the printer technologies are extremely different, and that 3D printers are an exciting piece of futuristic technology! **Rest of the lesson:** for the remainder of the lesson, the pupils will be split into groups of three (these groups will be decided later on), and given the task of designing and creating a new 3D shape. There should be some kind of context for it (e.g. what will it be used for?), and all the key terminology from previous lessons should be used. The groups may want to take inspiration from previous homework (e.g. the list of 3D shapes, what 3D shapes are found in nature…) and record the properties of their new shape, as well as try to create a suitable net for it (if this is possible). They should also think of a name for their created shape, which can use literary cues (tri/quad/pent/hex etc.). **Homework:** finish designing the 3D shape before lesson 6 – this can be done at home, at school etc. A "diary" of the design and creation process should be kept, and a short presentation must be prepared for lesson 6.
Pupil Assessment (include approach and expected outcome): The homework on printers will be discussed at the beginning of the lesson, but the majority of the design work carried out in lesson 5 will be peer assessed in lesson 6.

Lesson Plan

Lesson 6 – Presenting the new 3D shapes
Learning Objectives: • Learn to give careful, objective feedback on presentations made by other pupils • Discuss the main processes that go into designing a new 3D shape
Resources: • Voting sheets (to be designed at the beginning of this lesson) • WWW/EBI sheets • Module evaluation form
Activities: **Beginning of lesson:** as a whole class, decide upon the 3 or 4 most important aspects of oral presentations (e.g. clarity of speech, precise use of vocabulary, enthusiastic delivery, novelty, involving everybody in the group, a "punchline/main point" to the presentation…). Before the presentations begin, the voting cards will be created, with the discussed criteria written on them. **Rest of the lesson:** each group will take it in turns to give a short presentation (around 5 minutes) about the 3D shape they created. They should remember the key criteria, and try to create the best possible presentation. Each group's presentation will be judged on these key criteria, with each of the other groups able to give a score out of 10 for each component. Extra feedback will also be collected in the form of WWW/EBI sheets. **End of the lesson:** once all the presentations have been given, a final module evaluation form will be completed by everyone, which will bring a close to the main assessment. The group(s) which scored the highest will also be rewarded with small prizes!
Pupil Assessment (include approach and expected outcome): Pupils will be assessing each other during the presentations in the form of scores/feedback; the final module evaluation form will also be used to summarise their thoughts on the unit as a whole.