University Learning in Schools

Mathematics

3D Geometry in the real world

Module Outline

The Brilliant Club

Department for Education

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University Learning in Schools

Module outline

Title of Module: 3D Geometry in the real world
Teacher/researcher pair: Dr Joan Telford/Mr Simon Chu

Module outline
1. What is the overall topic area? The overall topic is 3D geometry, specifically the 3D platonic solids (tetrahedron, cube, octahedron, dodecahedron and icosahedron) as well as the Archimedean solids and all their properties (vertices/edges/faces and rotational symmetry).
2. How does it link to current research and why does the researcher think this is an important topic for pupils to get to grips with? The field of algebraic geometry can be restricted to certain dimensions, but even in the 2D and 3D cases many things can be complicated. One way of looking at algebraic shapes/structures is to look at their symmetry group, which ties into modern group theory (e.g. the Monster group and the Mathieu groups). In addition, there are objects such as modular forms which possess a lot of symmetry, but can be viewed from several angles: they can be thought of as algebraic objects or analytic functions. For 2D shapes, lines of (reflection) symmetry and centres of rotational symmetry can be found “by eye”, but for 3D shapes the concept of symmetry is a little trickier. The 3D world is something which is not covered in as much detail as the 2D world is in the national curriculum, yet we live our lives in three dimensions, so it is important to get a handle on how 3D shapes behave. This unit will provide pupils with lots of hands-on time working with 3D shapes (exploring their properties) and let them research their uses/applications in today’s society through the homework. The lessons will also emphasise some of the similarities and differences between the 2D and 3D worlds, using nets to act as a bridge.

3. How is this topic aiming to improve teacher subject knowledge?

This unit of work builds upon 2D geometry with 3D geometry, enabling teachers to see the similarities (e.g. we still have vertices/edges/faces) and the differences – for example, there's no such thing as "reflectional symmetry" in 3D, only rotational symmetry, which is a bit different even from the 2D version, because instead of a centre of rotation we have an *axis* of rotation.

The rotational symmetry in particular provides a way back into matrices and basic group theory. Group theory is sometimes introduced to year 13 pupils, but is normally reserved until university. This provides some background motivation for looking at rotational symmetry of the platonic solids, which turn out to be "nice" groups to study, such as A_4, S_4 and A_5.

Finally, the topic of Euler's formula often crops up when looking at 3D shapes, but its generalisation to non-convex 3D shapes is less well-known and relies on the idea of the *genus* and Euler characteristic; these are studied via the cohomology of an algebraic variety, which is a rich and fantastic area for further reading for teachers with lots of time on their hands!

4. What key texts/case studies/experiments/processes are being considered?

The key processes include:

- Properties of 2D shapes (vertices, edges, faces, angles, lines of symmetry, rotational symmetry)
- Properties of 3D shapes (vertices, edges, faces, axes of rotational symmetry)
- Nets – constructing 3D shapes from their nets, and deconstructing 3D shapes to form nets

Some case studies for reference include:

- "Students' 3D geometry thinking profiles", Pittalis et al., Department of Education, University of Cyprus
- "Creativity in three-dimensional geometry: how can an interactive 3D-geometry software environment enhance it?", Sophocleous and Pitta-Pantazi, Department of Education, University of Cyprus

The use of nets to act as a link between seeing a 3D shape and "flattening it out" as a 2D net will be very important, and is utilised across lessons 1 to 4.

5. How is this topic aiming to enhance pupils' subject knowledge and improve pupil outcomes?

Aside from reinforcing knowledge about 2D geometry and showing pupils the similarities and differences between 2D and 3D geometry (as mentioned previously), various key skills will be worked on.

For example, their spatial reasoning will be improved by going back and forth between the 2D and 3D world via nets, with the realisation that the same 3D shape (like an icosahedron) can have lots of different looking nets. The homework activities and the final two lessons ("design and create your own 3D shape") will let the pupils get creative and ask them to apply what they have learnt from the previous lessons. Added benefits of this are the team-working opportunities and presentations skills that they will get to practise.

Overall, it is hoped that by delving into the 3D world, we will provide some motivation for studying geometry further in the future, and we can show that the 2D and 3D universes are rich in structure.